Lecture 1

$y = f(x)$ is a function in one variable.

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\delta x) - f(x)}{\delta x} \quad \text{is called the odinary derivative of} \quad y = f(x)$$

$Z = f(x_1, x_2, \cdots, x_n)$ is a function in n variables.

$$\frac{d}{dx} f(x,y,z) = \frac{\partial T}{\partial x} + \frac{\partial T}{\partial y} + \frac{\partial T}{\partial z} \quad \text{is called the partial derivative}$$

For plotting $Z = f(x,y)$ we require 3D space and we obtain a surface.

For $Z = f(x_1, x_2, x_3)$ we require 4 dimensional space.

Thus a function in n variables $Z = f(x_1, x_2, \cdots, x_n)$ we require $n+1$ dimensional space.

Limit of a function

Let $Z = f(x,y), \quad (x,y) \in \mathbb{R}^2$

$f : \mathbb{R}^2 \to \mathbb{R}$

$$\lim_{(x,y) \to (x_0,y_0)} f(x,y) = L$$

For every $\epsilon > 0$ there exists $\delta > 0$ such that $|f(x,y) - L| < \epsilon \quad \forall \quad (x,y)$ and $0 < |(x,y) - (x_0,y_0)| < \delta$.

In other words, if $\lim_{(x,y) \to (x_0,y_0)} f(x,y) = L$, there exits a δ ball of (x_0, y_0) such that $f(x,y)$ lies in the epsilon neighborhood of L for every $\epsilon > 0$.

To prove limit does not exist, we show that atleast two paths exists on which values of limit is different.

Ex: $Z = f(x,y) = \frac{(x+y)^2}{x^2+y^2}$

Lets take (x_0, y_0) = (0, 0)

Approaching from path 1 : (first y → 0, then x → 0)

$$\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{x\to 0}\left\{\lim_{y\to 0} f(x,y)\right\}$$

$$= \lim_{x\to 0}\{1\} = 1$$

Approaching from path 2: (along the line y = x)
as x → 0, y → 0

$$\lim_{x\to 0} f(x,y) = \lim_{x\to 0}\frac{(2x)^2}{2x^2} = 2$$

Similarly on path 3: (along y = -x)

$$\lim_{x\to 0} f(x,y) = \lim_{x\to 0}\frac{0}{2x^2} = 0$$

⇒ Limit does not exists at (0, 0).

#semester-1 #mathematics #calculus