Lecture 8

Norm function

- $||\cdot||$ is called the norm function and $||X||$ is called the norm of X.
- $||\cdot||_2$ is called euclidean norm. $||X||_2 = \sqrt{\sum_{i=1}^{n} |X_i|^2}$

If $P = \begin{bmatrix} \uparrow & \uparrow & & \uparrow \\ X_1 & X_2 & \cdots & X_n \\ \downarrow & \downarrow & & \downarrow \end{bmatrix}$ then $P^T = \begin{bmatrix} \leftarrow & X_1 & \rightarrow \\ \leftarrow & X_2 & \rightarrow \\ & \vdots & \\ \leftarrow & X_n & \rightarrow \end{bmatrix}$.

$$\Rightarrow P^TP = \begin{bmatrix} \leftarrow & X_1 & \rightarrow \\ \leftarrow & X_2 & \rightarrow \\ & \vdots & \\ \leftarrow & X_n & \rightarrow \end{bmatrix} \begin{bmatrix} \uparrow & \uparrow & & \uparrow \\ X_1 & X_2 & \cdots & X_n \\ \downarrow & \downarrow & & \downarrow \end{bmatrix}$$

$$= \begin{bmatrix} X_1^TX_1 & X_1^TX_2 & \cdots & X_1^TX_n \\ X_2^TX_1 & X_2^TX_2 & \cdots & X_2^TX_n \\ \vdots & \vdots & \ddots & \vdots \\ X_n^TX_1 & X_n^TX_2 & \cdots & X_n^TX_n \end{bmatrix}$$

But when A is real symmetrix matrix, $X_i^TX_j = 0$ for $i \neq j$.

$$\Rightarrow P^TP = \begin{bmatrix} X_1^TX_1 & 0 & \cdots & 0 \\ 0 & X_2^TX_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & X_n^TX_n \end{bmatrix}$$

Now,

$$X_1^TX_1 = [x_1 \quad x_2 \quad \cdots \quad x_n] \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = x_1^2 + x_2^2 + \cdots + x_n^2$$

$$\Rightarrow X_1^TX_1 = ||X_1||^2$$

$$or$$

$$||X_1|| = \sqrt{X_1^T X_1}$$

For making $P^T P = I$ (in order to make matrix P orthogonal) we modify X as $\frac{X}{||X||}$ called the normalized vector P.

Thus $P = \begin{bmatrix} \uparrow & \uparrow & & \uparrow \\ \frac{X_1}{||X_1||} & \frac{X_2}{||X_2||} & \cdots & \frac{X_n}{||X_n||} \\ \downarrow & \downarrow & & \downarrow \end{bmatrix}$.

We will get $P^T P = I$ now as $X_1^T X_1 = ||X_1||^2$.
Now we can have $P^{-1} = P^T$.

$$P^{-1} AP = D \Rightarrow P^T AP = D$$

This way diagonalization process of a matrix can be made simple.

#semester-1 #mathematics #matrices