Lecture 1

Vector spaces & Matrix transformation.

Matrix operations can be viewed as transformation of vector spaces of one dimension to some other.

For example if we have Y = AX, where

$$A_{m,n} = \begin{bmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,n} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m,1} & a_{m,2} & \cdots & a_{m,n} \end{bmatrix}$$

we can say that $A_{m,n}$ is transformation from $\mathbb{R^n} \to \mathbb{R^m}$
If,

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

the mapping will be called identity mapping. The shape in Y-plane will be same as on the X-plane.
If for k > 1,

$$A = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$$

the mapping will be called stretching map. The shape in Y-plane will be twice in each dimension as on the X-plane. (If the shape is a line it will be twice as long and if the shape is a square, it would have 4 times the original area)
Similarly, for k < 1 it will be compression mapping and for k < 0, the direction of vector become opposite.

If,

$$A = \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix}$$

$A : \mathbb{R}^2 \to \mathbb{R}^2$ the vector would rotate θ degrees in anticlockwise direction.

$$y_1 = rcos(\theta + \psi) = r(cos\theta cos\psi - sin\theta sin\psi)$$

$$y_2 = rsin(\theta + \psi) = r(sin\theta cos\psi + sin\psi cos\theta)$$

So, substituting value of x_1 and x_2 we get

$$y_1 = x_1 cos\theta - x_2 sin\theta$$

$$y_2 = x_1 sin\theta + x_2 cos\theta$$

Thus in matrix format we can write them as:

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

For reflection mapping (about x-axis), $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$.

For reflection mapping (about y-axis), $A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$.

Eigen Values & Eigen Vectors

Eigenvectors are vectors for which Y = AX is parallel to X. In other words: AX = λX.

In this equation, X is an eigenvector of A and λ is an *eigenvalue* of A.

#semester-1 #mathematics #matrices